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Jason Reed, CFA, FCIA

The St. Petersburg Paradox

I was recently reminded of a classic decision theory problem involving the following hypothetical casino game. The pot starts at $1. A fair coin is repeatedly tossed, and the pot is doubled after each successive head (H). The game ends on the first tail (T) and the player wins whatever is in the pot at that time. The pattern of possible outcomes is outlined in the table below.

Sequence	Winnings
T	$1
H, T	$2
H, H, T	$4
H, H, H, T	$8
… etc	

If the player is lucky enough to see thirty consecutive heads, the pot grows to over $1 billion. What is the maximum amount you'd pay to play this game? There's no right or wrong answer, as everyone has their own level of risk aversion and utility for wealth. In fact, although the expected value of the payoff is infinite, most people are willing to pay no more than a token amount to play the game. This is known as the St. Petersburg Paradox.

If we were to think of the St. Petersburg game as a stock, the growth rate of earnings and the valuation discount rate are both 100%. The game has infinite expected value because the two rates are equal. The expected value (payoff x probability) of the 30th toss is the same as the first toss, despite the infinitesimal chances of surviving that far into the game. In the world of investing, we are currently seeing investors place a high premium on fast growing businesses, as they did during the late 1990s. This is due in part to growth expectations that are approaching, or even exceeding, valuation discount rates. In the St. Petersburg game, people intuitively discount the possibility of a long run of growth.

There are many ways in which growth can be interrupted for equity investors. An emerging competitor might build a better product or service. Management might make operational errors or poor re-investment decisions, including unwise acquisitions. A large and fast-growing business could be regulated into mediocrity. Finally, it is simply unsustainable for very large businesses to grow much faster than the broader economy. For all these reasons, high growth rates tend to have more downside risk than upside potential and should therefore not be expected to persist for too long.

The table below illustrates the payoffs of the St. Petersburg gamble as a multiple of the initial pot. Expressed in this format, they are analogous to price to earnings (P/E) ratios for stocks. For illustrative purposes, the table also considers slight variations in the growth rate of the payoff, and the probability of the coin landing tails (a rising probability correlates to a higher discount rate).

St. Petersburg Payoff Sensitivity to Game Parameters

	Probability of Tails			
Growth Rate	50%	51%	52%	53%
100%	Infinite	25.5x	13.0x	8.8x
98%	50.0x	17.1x	10.5x	7.6x
96%	25.0x	12.9x	8.8x	6.7x
94%	16.7x	10.3x	7.6x	6.0x

Starting at the red box (a relatively high P/E ratio of 25.5x), we can see that small reductions in the growth rate (moving down one box) or small increases in the discount rate (moving right one box), or both (moving diagonal), cause losses between 30% and 60%. In general, unexpected reductions in growth cause losses that are potentially magnified by rising discount rates. When starting from the green box (a lower P/E ratio), losses are reduced to between 20% and 40%.

As government bond yields decline, and even go negative in some parts of the world (e.g. Japan, Europe), investors must contemplate the investment version of the St. Petersburg Paradox. The premium for fast growing businesses has grown to levels not seen since the tech bubble that inflated during the 1990s. As we know, expectations for profitable growth never materialized and the bubble deflated. Today we have the added risk of even lower interest rates than those that helped fuel the tech bubble.

The St. Petersburg game is called a paradox because of the gap between the theoretical expected value and what people are practically willing to pay. In investing, we resolve the paradox by insisting on a margin of safety in the price we pay for businesses. Without it, investing looks too much like gambling.

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